Dualité

- F sev de E tq $\exists G, G'$ ~~de~~ tq $F \oplus G' = E = F \oplus G$ alors $G \simeq G'$
- G de dim $< +\infty$. $F \oplus G = E$. G' de dim $=$ dim G et $F \cap G' = \{0\}$

  $F \oplus G' = E$.

- H hyperplan $\Leftrightarrow \exists u \in E \setminus H$ tq $H \oplus \mathbb{K}u = E$
- F sev, H hyperplan. codim $F \cap H = \begin{cases} d \text{ si } F \subset H \\ d+1 \text{ sinon} \end{cases}$ (codim $F = d$)
- V de codim p. $\exists H_1, \dots, H_p$ HP tq $V = \bigcap_{i=1}^{p} H_i$
- H hyperplan, $\exists \varphi \in E^*$ tq $H = \operatorname{Ker} \varphi$.

$\to \forall \varphi \in E^*$ $\operatorname{Ker}\varphi$ HP.

Thm fondamental de la dualité

$\varphi_1, \dots, \varphi_p \in E^*$. $\psi \in E^*$

$$\psi \in \operatorname{Vect}(\varphi_1, \dots, \varphi_p) \Longleftrightarrow \bigcap_{i=1}^{p} \operatorname{Ker}\varphi_i \subset \operatorname{Ker}\psi.$$

$\operatorname{Ker}\varphi_1 \subset \operatorname{Ker}\varphi_2 \Rightarrow \varphi_1 \propto \varphi_2$.

$\Rightarrow$ codim $\bigcap_{i=1}^{p} \operatorname{Ker}\varphi_i = \operatorname{rg}_{E^*}(\varphi_1, \dots, \varphi_p)$

$\to$ À toute base du dual E^* est associé une unique base de E

$e_i \leftrightarrow e_i^*$